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ON THE DYNAMIC STABILITY OF ECCENTRICALLY REINFORCED CIRCULAR CYLINDRICAL SHELLS

By

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p. 3. bottom line monocoque

p. 6. 3rd eq. of (6) $M_{xy}^{(m)} = - M_{yx}^{(m)} = D_G \varkappa_{xy}$

p. 7. eq. (9): $N_x^{(S)} = \frac{E_S A_S}{d} \varepsilon_x - \frac{E_S A_S \bar{z}_S}{d} \varkappa_x$

p. 7. eq. (10): $N_y^{(R)} = \frac{E_R A_R}{\ell} \varepsilon_y - \frac{E_R A_R \bar{z}_R}{\ell} \varkappa_y$

p. 11. eqs. (18): $M_x = F_{Sb} \varepsilon_y - D_{HS} \varkappa_x - D_\nu \varkappa_y$

$$M_y = F_{Rb} \varepsilon_y - D_\nu \varkappa_x - D_{HR} \varkappa_y$$

p. 12. Matrix of (20): $\underset{\sim}{\begin{pmatrix} K_{HS} & K_\nu & 0 & -F_{Sb} & 0 & 0 \\ K_\nu & K_{HR} & 0 & 0 & -F_{Rb} & 0 \end{pmatrix}}$

p. 22. eq. (50) first line: $-\left[\left(D_{HS} - \frac{K_H F_{Sb}^2}{K_{HR} K_{HS} - K_\nu^2}\right) w_{,xxxx} + 2\left(D_2 + \frac{K_\nu F_{Sb} F_{Rb}}{K_{HR} K_{HS} - K_\nu^2}\right) w_{,xxyy}\right.$

p. 23. last of (54) $D_{12} = D_2 + \frac{K_\nu F_{Sb} F_{Rb}}{K_{HR} K_{HS} - K_\nu^2}$

p. 27. 2nd column $D_{12} = D_2 + \frac{K_\nu F_{Sb} F_{Rb}}{K_{HR} K_{HS} - K_\nu^2}$

1. Introduction

Since the last report [1] considerable progress has been made toward a solution of the dynamic stability of an eccentrically reinforced cylindrical shell. In order to solve a specific problem, the ends of the eccentrically reinforced cylindrical shell are controlled to approach each other at a specific rate $V = V(t)$. This is similar to the first-known dynamic stability investigation of the column [2] in which Hoff used a constant $V = V_o$. This controlled end approach causes an axial loading whose dynamic response is investigated.

2. Method of Solution

The dynamic equilibrium and compatibility equations were derived as equations (55) and (56) in [1]. In the sequel, these equations will be called field equations. The directions of z and w have been reversed from that used in [1]. Similarly, the moments and stress resultants comply now with those used by Timoshenko [3], or Volmir [4] (except for N_{xy} and N_{yx} in Volmir's book) and the thickness of the monocoque shell is h, leaving t for time. The field equations are easily modified to include initial imperfections. Denoting by $w_{(1)}$ the total radial displacement by $w_{(o)}$ the radial displacement due to initial imperfections and by Φ the stress function when initial imperfections are considered, the field equations become:

$$D_{11}\left[w_{(1),xxxx} - w_{(0),xxxx}\right] + 2D_{12}\left[w_{(1),xxyy} - w_{(0),xxyy}\right] + S_{11}\Phi_{,xxxx} - 2S_{12}\Phi_{,xxyy} + S_{22}\Phi_{,yyyy} - \Phi_{,xx} w_{(1),yy} + 2\Phi_{,xy} w_{(1),xy} - \Phi_{,yy} w_{(1),xx} - \frac{\Phi_{,xx}}{R} - p + \bar{m} w_{(1),tt} - I_{\bar{m}}\left[w_{(1),xxtt} + w_{(1),yytt}\right] = H = 0 \qquad (1)$$

$$A_{11}\Phi_{,xxxx}+2A_{12}\Phi_{,xxyy}+A_{22}\Phi_{,yyyy}-S_{11}[W_{(1),xxxx}-W_{(0),xxxx}]+2S_{12}[W_{(1),xxyy}-W_{(0),xxyy}] \quad (1)$$
$$-S_{22}[W_{(1),yyyy}-W_{(0),yyyy}]-W_{(1),xy}^2+W_{(1),xx}W_{(1),yy}+W_{(0),xy}^2-W_{(0),xx}W_{(0),yy}+\frac{1}{R}[W_{(1),xx}-W_{(0),yy}]=0$$

where the comma notation is used for partial differentiation, and where the A's, S's, and $\bar{m}$ are defined as in [1] $(t \to h)$. p is the external lateral pressure (assumed zero in the sequel) and the rotatory inertia effects are included. $I_{\bar{m}}$ is defined by

$$I_{\bar{m}} = \frac{\rho h^3}{12} + \rho_S \frac{I_{SC}+A_S\bar{z}_S^2}{d} + \rho_R \frac{I_{RC}+A_R\bar{z}_R^2}{\ell} \quad (2)$$

The radial displacements are then chosen like those in reference [5], where the dynamic stability of the monocoque cylindrical shell is investigated.

$$W_{(0)} = f_0 \sin\alpha x \sin\beta y + g_0 \sin^2\alpha x \sin^2\beta y$$
$$W_{(1)} = f_1(t)\sin\alpha x \sin\beta y + g_1(t)\sin^2\alpha x \sin^2\beta y \quad (3)$$

where

$$\alpha = \frac{m\pi}{L} = \frac{\pi}{a}$$
$$\beta = \frac{n}{R} = \frac{\pi}{b} \quad (4)$$

m is the number of half wave lengths, a, (ma = L) in the axial direction and n is the number of full wave lengths, 2b, $(n2b = 2\pi R)$ in the circumferential direction. "Spatial harmony" is assumed between the initial imperfection and the total radial displacement. The first term of (3) allows for a "checkerboard" - or chess board,- and the second for a "diamond" buckling pattern.

The above radial displacement assumption does not exactly satisfy neither simply-supported, nor clamped boundary conditions. It can be shown, however, that clamped boundary conditions are satisfied on the average over the circumference.

On substituting (3) into the second equation of (1), and integrating, a stress function Φ (x,y,t) is obtained in the form:

$$\Phi(x,y,t) = \Phi_h - \bar{N}_{ox}\frac{y^2}{2} - \bar{N}_{oy}\frac{x^2}{2} \tag{5}$$

where Φ_h corresponds to the homogeneous problem, $\bar{N}_{ox}$ and $\bar{N}_{oy}$ are due to the axial loading. Φ_h is given by:

$$\begin{aligned}\Phi_h = {} & \lambda_1 \sin\alpha x \sin\beta y + \lambda_2 \sin 3\alpha x \sin\beta y + \lambda_3 \sin\alpha x \sin 3\beta y + \lambda_4 \cos 2\alpha x \\ & + \lambda_5 \cos 2\beta y + \lambda_6 \cos 2\alpha x \cos 2\beta y + \lambda_7 \cos 4\alpha x + \lambda_8 \cos 4\beta y \\ & + \lambda_9 \cos 4\alpha x \cos 2\beta y + \lambda_{10} \cos 2\alpha x \cos 4\beta y\end{aligned} \tag{6}$$

The λ's are rather involved expressions that contain f_1, f_o, g_1, g_o, α, β, A's, S's. The detailed listing will be forthcoming in the final report.

In the treatment of the monocoque shell in reference [5], $\bar{N}_{oy}$ is assumed as the membrane hoop stress resultant due to hydrostatic pressure. In the absence of the latter, no Poisson interaction is therefore possible. In contrast to such a simplification, $\bar{N}_{ox}$ and $\bar{N}_{oy}$ are related by the closure condition of the shell which can be stated as:

$$\bar{N}_{oy} = \frac{A_{13}}{A_{11}}\bar{N}_{ox} - \frac{\beta^2}{8A_{11}}\left[(f_1^2 - f_o^2) + \frac{3}{4}(g_1^2 - g_o^2)\right] + \frac{g_1 - g_o}{4A_{11}R} \tag{7}$$

$\bar{N}_{ox}$ is interpreted physically as the axial stress resultant at the ends of the shell when averaged over the circumference.

The rate at which the ends of the cylindrical shell approach each other is assumed in the form

$$V(t) = V_0 e^{-\gamma t} \tag{8}$$

which reduces to that of Hoff [2] for $\gamma = 0$.

The axial loading results from the mean end shortening $\bar{e}$. given by

$$\bar{e} = -\frac{1}{2\pi R}\int_0^{2\pi R}\int_0^{L} u_{,x}\,dx\,dy = \int_0^{t} V(\tau)\,d\tau \tag{9}$$

With equations (8) and (9), $\bar{N}_{ox}$ can be shown to be:

$$\bar{N}_{ox} = \frac{V_0(1-e^{-\gamma t})}{\gamma L\left(A_{22}-\frac{A_{13}^2}{A_{11}}\right)} - \frac{1}{8}\frac{\alpha^2+\frac{A_{13}}{A_{11}}\beta^2}{\left(A_{22}-\frac{A_{13}^2}{A_{11}}\right)}\left[(f_1^2-f_0^2)+\frac{3}{4}(g_1^2-g_0^2)\right]+\frac{1}{4R}\frac{A_{13}}{A_{11}}\frac{g_1-g_0}{\left(A_{22}-\frac{A_{13}^2}{A_{11}}\right)} \tag{10}$$

On using equations (7) and (10) in (5), the stress function is obtained in terms of the known quantities.

The resulting stress function expression is then introduced into the equilibrium equation of (1). Equilibrium is being satisfied in the mean by using the Bubnov-Galerkin method, e.g.

$$\int_0^{L}\int_0^{2\pi R} H \sin\alpha x \sin\beta y\,dx\,dy = 0$$

$$\int_0^{L}\int_0^{2\pi R} H \sin^2\alpha x \sin^2\beta y\,dx\,dy = 0 \tag{11}$$

Space limitation does not allow to list the expression for H in detail. It can be easily appreciated that H would cover several pages and will be given in the forthcoming final report.

Carrying out the Galerkin procedure leads to the following pair of second-order coupled ordinary differential equations of the third degree:

$$\frac{d^2f_1}{dt^2} = B_1 f_1 + B_2 g_1 + B_3 f_1 g_1 + B_4 f_1 g_1^2 + B_5 f_1^3 + B_6 f_1 \frac{1-e^{-\gamma t}}{\gamma} + B_7 \tag{12}$$

$$\frac{d^2g_1}{dt^2} = C_1 g_1 + C_2 f_1 + C_3 g_1^2 + C_4 f_1^2 + C_5 f_1^2 g_1 + C_6 g_1^3 + C_7 g_1 \frac{1-e^{-\gamma t}}{\gamma} + C_8 \frac{1-e^{-\gamma t}}{\gamma} + C_9$$

The B's and C's are rather involved constants. Space does not allow a detailed listing and reference is made to the forthcoming final report. The B's and C's contain α, β, A's B's, f_o, g_o, $\bar{m}$, R, L, γ, V_o.

On deleting the diamond pattern amplitude $g_1(t)$, setting $\gamma = 0$, taking the B's and C's for the monocoque shell, letting $R \to \infty$, m = 1, $\nu \to 0$ etc., the above equations reduce to the single equation used by Hoff [2] for the column except for a minor factor.

The reduced equation becomes:

$$\frac{d^2f_1}{dt^2} = B_1^{(c)} f_1 + B_5^{(c)} f_1^3 + B_6^{(c)} f_1 t + B_7^{(c)} \tag{13}$$

where

$$\left.\begin{aligned} B_1^{(c)} &= -\left(\frac{\pi}{L}\right)^4 \frac{EI}{\rho A} + \frac{3}{16}\left(\frac{\pi}{L}\right)^4 \frac{E f_o^2}{\rho} \\ B_5^{(c)} &= -\frac{3}{16}\left(\frac{\pi}{L}\right)^4 \frac{E}{\rho} \\ B_6^{(c)} &= \left(\frac{\pi}{L}\right)^2 \frac{E V_o}{\rho L} \\ B_7^{(c)} &= \left(\frac{\pi}{L}\right)^4 \frac{E I f_o}{\rho} \end{aligned}\right\} \tag{14}$$

(14)

and where the superscript (c) implies column. If the factor 3/16 is replaced by 1/4, Hoff's equation for the simply supported column is obtained. This slight discrepancy has only a minor effect since 3/16 multiplies the imperfection term of $B_1^{(c)}$. Its effect on $B_5^{(c)}$ is multiplied by f_1^3, which becomes significant only for large f_1.

Once a solution of (12) has been accomplished, $\bar{N}_{ox}$ can be calculated from (10).

A computer program has been developed that will calculate the B's and C's for a given reinforced shell and assumed m and n. It integrates the equation by means of the Runge-Kutta method. The latter method was used in reference [5]. The method has been tested successfully on the constant coefficient reduced linear systems of (12) which is amenable to a closed form solution. To each pair m, n, there corresponds a $\bar{N}_{ox} = \bar{N}_{ox}(t)$.

For a particular mode (m,n) there exists a $\bar{N}_{ox\ max}$ which might be called the critical dynamic buckling load for that mode. Which of these possible modes (m,n) is the true mode, remains to be answered.

3. Related Work

In reference [5] m = n is assumed, and the criterion is adopted that, the curve $\mathfrak{f} = (f_1 + g_1)/h = \mathfrak{f}(m,n,t)$, which departs earliest from the time-axis and also is first to reach its peak, is the proper curve, so that m = n is determined. From the diagrams of reference [5], it appears, however, that $\mathfrak{f}$ departs earlier and earlier with increasing n = m, so that this criterion remains inconclusive.

In Agamirov and Volmir's paper [6], which is one of the earliest in the field of dynamic buckling of shells, the dynamic stability of a monocoque shell due to pressure loads (ramp) is investigated, using a similar approach. The authors assume m = 1 from the very beginning and set $f_1 = g_1$. They are faced with the same dilemma which they resolve by taking again the n that corresponds to the curve $f_1 = f_1(n,t)$ which departs earliest from the time axis.

4. Present Status and Future Work

Preliminary numerical solutions have been obtained for the stringer shell labeled number 1 in Card's report [7] for a constant rate of end approach $V_o = 100$ in/sec. These results are presently reviewed particularly with respect to the modal numbers m,n. So far, no satisfactory criterion has been found in choosing the proper m and n. A linear Donnell-type static equation has been derived which leads to the following expression for the static axial buckling load:

$$N_x = \frac{\Omega}{\theta[\theta^2 A_{11} + 2\theta A_{12} + A_{22}]}\Big\{\theta^4\mu_1 + \theta^3\mu_2 + \theta^2\mu_3 + \theta\mu_4 + \mu_5 + \frac{2}{R}\frac{\theta}{\Omega}(\theta^2 S_{11} - 2\theta S_{12} + S_{22}) + \frac{\theta^2}{\Omega^2 R^2}\Big\} \tag{15}$$

where

$$\left.\begin{aligned}
\Omega &= \beta^2 = \left(\frac{n}{R}\right)^2\\
\theta &= \gamma^2 = \left(\frac{b}{a}\right)^2 = \left(\frac{\pi R}{L}\right)^2\left(\frac{m}{n}\right)^2\\
\mu_1 &= A_{11}D_{11} + S_{11}^2\\
\mu_2 &= 2(A_{12}D_{11} + A_{11}D_{12} - 2S_{11}S_{12})\\
\mu_3 &= A_{22}D_{11} + 4A_{12}D_{12} + 2(S_{11}S_{22} + 2S_{12}^2)\\
\mu_4 &= 2(A_{22}D_{12} + A_{12}D_{22} - 2S_{12}S_{22})\\
\mu_5 &= A_{22}D_{22} + S_{22}^2
\end{aligned}\right\} \tag{16}$$

If one chooses m = 2 n = 6 as reported in [7], the resulting buckling loads for the internally and the externally stiffened cylinder agree quite closely with Card's results. (see following Table)

Defining,

$$\eta = \frac{N_{x\,EXT.}}{N_{x\,INT.}} \tag{17}$$

it can be seen from the results in the table below that η depends to a considerable extent on the choice of m,n.

m	n	$N_{x\ int}$ [lb/in]		$N_{x\ ext}$ [lb/in]		η	ν
		EQ.(16)	CARD'S TEST	EQ.(16)	CARD'S TEST	EQ.(16)	
1	5	706		1176		1.67	0.1579
1	6	800		1138		1.42	0.1316
2	6	849	800	1928	1875	2.27	0.2632
2	7	755		1610		2.14	0.2256

It is quite easy to determine a function $\eta = \eta(\nu)$ from equation (15), for a fixed n. The maximum of η of all these n- parameter curves lies within $0.3 < \nu < 0.4$ and has an absolute maximum for n = 7 for Card's stringer shell. From these considerations, it appears that any reliable theoretical prediction of the static or dynamic critical buckling load would have to include a determination of m and n. It is recalled that the linear classical axial buckling analysis leaves the mode m, n undetermined. Non-linear analysis will yield m,n only if a variation of the parameters includes the half wave lengths, or other parameters that are related to m,n.

References

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